Lecture 6: Radiative transfer and radiative forcing

In this lecture, we will learn how increasing CO_2 affects the planetary energy budget, or specifically why doubling CO_2 causes

- more-or-less the same reduction in outgoing irradiance (or brightness temperature) over a range of wavelengths regardless of the concentration of CO_2 to start with, and
- very little change in outgoing irradiance over the rest of the spectrum.

Let's start by re-iterating how it doesn't work. Consider the standard simple "greenhouse" model:

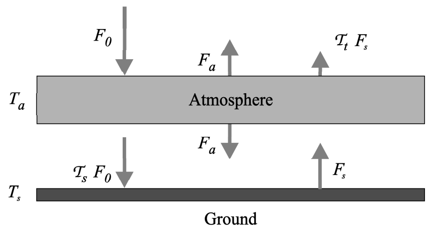

Figure 1: A very simple model of the greenhouse effect (from Andrews, 2000, with apologies).

Balancing fluxes, assuming that the system is in equilibrium (which is OK) and that no other forms of heat transport are significant (which is not), gives

$$F_s = \sigma T_s^4 = F_0 \frac{(1+\mathcal{T}_{\rm solar})}{(1+\mathcal{T}_{\rm thermal})} \, .$$

If we compute $\mathcal{T}_{\rm thermal}$ for the tropical, clear-sky atmosphere using MODTRAN, we find

$\mathcal{T}_{\rm thermal} = 0.1456$ for CO_2=375ppm

which implies $T_s = 290$K, remarkably close to observed.

This coincidence gives many people the illusion that this simple model captures the essence of the greenhouse effect. But they are wrong: re-do the MODTRAN calculation after doubling CO_2 and you find

$\mathcal{T}_{\rm thermal} = 0.1427$ for CO_2=750ppm

Implying that doubling CO_2 increases T_s^4 by 0.25%, or T_s by < 0.2K.

In fact, the simple greenhouse model is even more misleading than this, because transmittances are even lower in the wavelengths actually affected by CO_2.

There is a message here: models can get present-day climate right for the wrong reasons, and hence make wildly inaccurate predictions of climate change. This doesn't just apply to one-line models.

So, how can Al Gore (and almost everyone else) have got it so wrong?

The essential physics of the enhanced greenhouse effect due to increasing CO_2 arises from three facts:

- CO_2 number density, and with it optical depth at the wavelengths affected by CO_2, decreases approximately exponentially with height.
- Temperatures decrease linearly with height in the troposphere (the lapse rate).
- There are infra-red wavelengths in which the stratosphere is nearly transparent ($\mathcal{T} \simeq 0$, or "optically thin") but the atmosphere as a whole is nearly opaque ($\mathcal{T} \simeq 1$, or "optically thick").

We'll start with a qualitative explanation, and then work it through in more detail.

- At frequencies ν strongly absorbed by CO_2, outgoing spectral irradiance I_ν emanates not from the surface, but from altitudes at which the optical depth χ_ν, or the density of the absorber ρ_a, becomes low enough for photons to escape to space.
- We defined optical depth in the previous lecture as the negative logarithm of the transmittance from altitude z to space:

$$\chi_\nu(z) = -\ln(\mathcal{T}_\nu(z,\infty))$$

In the problem sets, you will show (using Lambert's Law, see below) that

$$\frac{d\chi_\nu}{dz} = -k_\nu(z)\rho_a(z)$$

where k_ν is the absorption coefficient, which is a function of z because it depends on T and p.

- Because CO_2 is well-mixed, ρ_a decreases exponentially with the pressure scale height H.

- For a well-mixed absorber in an isothermal atmosphere with an absorption coefficient proportional so some power of p ("pressure broadening"), optical depth decreases exponentially with height

$$\chi_\nu(z) = H_\chi k_\nu(0)\rho_a(0)e^{-z/H_\chi}$$

where $k_\nu(0)$ and $\rho_a(0)$ are the absorption coefficient and absorber density at the surface and H_χ is the scale height for optical depth. k_ν also depends on temperature, but this is a secondary effect in this problem.

- At the wavelengths that matter, $k_\nu \propto p$, so $\chi_\nu(z)$ decreases exponentially with scale height $H_\chi \simeq H/2$ (you will check this in the problem sets).
- $T(z)$ decreases linearly with the tropospheric lapse rate Γ until we reach an isothermal radiative-equilibrium stratosphere:

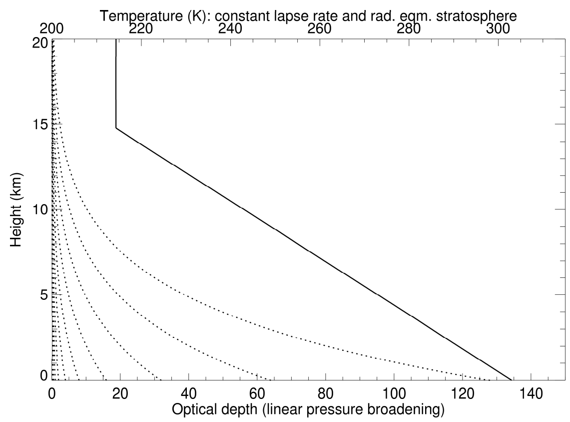

Figure 2: Schematic temperature profile compared with various optical depth profiles for various values of χ_s, the optical depth to the surface, corresponding to different wavelengths or CO_2 number density.

- Doubling the atmospheric concentration of CO_2 displaces $\chi_\nu(z)$ upwards by $H_\chi \ln(2)$ at all altitudes.

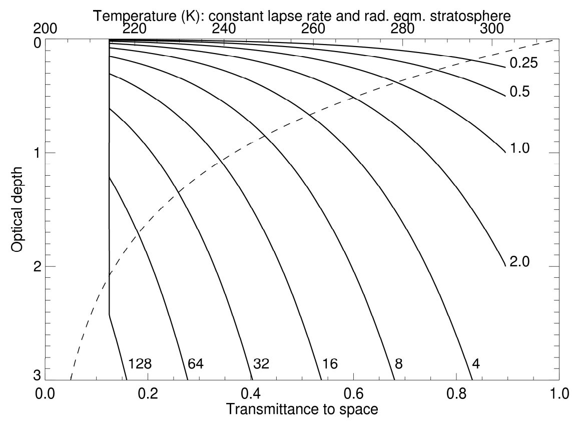

Figure 3: Solid lines show temperature profile as a function of optical depth, $T(\chi)$, for various values of χ_s. Dashed line and lower axis shows $\mathcal{T}(\chi_s)$

- If you were looking down on the atmosphere from above in the infra-red, you would not "see" altitude: you would see temperature as a function of optical depth, like this.
- Doubling CO_2 displaces $T(\chi_\nu)$ leftwards (towards colder temperatures) by $\Gamma H_\chi \ln(2)$.

- Optical depth to the surface ranges from <0.5 in "window" regions of the spectrum to many hundreds in strongly absorbed wavelengths. Note that $(k_\nu(0)\rho_a(0))^{-1} = H_\chi/\chi_s$ is the attenuation length scale at the surface, which ranges from > 10km to < 10m, depending on the wavelength ν .
- Temperatures at the altitude at which χ_ν becomes low enough for photons to escape to space fall by $\Gamma H_\chi \ln(2)$, reducing I_ν, provided this altitude is not (a) at the surface or (b) in the stratosphere.
- Plugging in numbers for the tropospheric lapse rate and assuming $H_\chi = H/2$ implies a doubling of CO_2 would have the same impact on I_ν as a 15-20K tropospheric cooling, which is close to the amount by which brightness temperatures fall in the wings of the CO_2 band in the MODTRAN calculation.
- *If* applied across the full spectrum, an 18K cooling of T_e would reduces outgoing energy flux by over 50Wm^{-2} (check) *but* (fortunately) the effect only

applies to a small fraction of the spectrum.

- Over most of the CO_2 absorption band, increasing CO_2 has very little impact on outgoing I_ν, because most outgoing irradiance emanates from the lower stratosphere (apart from a small spike right in the centre of the band where we are "seeing" higher temperatures in the mid-stratosphere). Likewise far away from the absorption band, increasing CO_2 has no impact. All the impact is in the "wings" of the band, where CO_2 is partially absorbing.
- Detailed radiative transfer calculations, also accounting for CO_2-induced cooling of the stratosphere and the interaction between CO_2 absorption and other greenhouse gases and clouds, give the *radiative forcing* (perturbation downward energy flux at the tropopause) due to doubling CO_2 as around 3.7Wm^{-2}.

The above argument should be enough to reassure (some) non-physicists that the concept of an anthropogenic greenhouse effect from increasing CO_2 is not just a eco-socialist conspiracy, but it relies on the concept of an "effective emission height", which isn't particularly satisfactory since emission is continuous. So we'll now do this calculation a bit more rigorously.

Atmospheric radiative transfer

Consider monochromatic radiance L_ν passing upwards vertically through a thin slab of atmosphere, thickness δz, containing a single radiatively active gas with mass absorption coefficient k_ν, density ρ_a. Lambert's law of absorption states:

$$dL_{\nu\,\mathrm{absorption}} = -k_\nu \rho_a L_\nu dz \ ,$$

while Kirkhoff's law of emission states:

$$dL_{\nu\,\mathrm{emission}} = k_\nu \rho_a B_\nu(T) dz \ ,$$

$B_\nu(T)$ being the Planck function.

Recalling the definition of optical depth and transmittance

$$\chi_\nu = -\ln(\mathcal{T}_\nu) \qquad \text{and} \qquad \mathcal{T}_\nu = \frac{L_{\nu 0}}{L_\nu}$$

where $L_{\nu 0}$ is the "top of atmosphere" (TOA) radiance (remember we're talking about *upward* radiance here) gives

$$d\chi_\nu = -\frac{d\mathcal{T}_\nu}{\mathcal{T}_\nu} = \frac{dL_\nu}{L_\nu} = -k_\nu \rho_a dz \qquad ,$$

(n.b.: if dz is positive, $d\chi$ is negative).

Applying Kirkhoff's law gives *Schwarzschild's equation* for radiance in the direction of decreasing χ (problem set 2):

$$-\frac{dL_\nu}{d\chi_\nu} + L_\nu = B_\nu \qquad .$$

The same equation applies to downward, "up-χ", radiance but without the minus sign.

Diffuse versus direct-beam radiation

So far, we've talked about vertical radiation only, which applies to attenuation of incoming solar radiation when the sun is overhead or a hypothetical upward-pointing light source. Infrared radiation is typically "diffuse", taking multiple optical paths up through the atmosphere.

It turns out that upward energy flux is governed by a very similar equation to Schwartzschild's, with B replaced with πB to account for integration over solid angle, and χ increased by a factor of 1.66 to account for multiple optical paths:

$$-\frac{dI_\nu}{d\chi_\nu} + I_\nu = \pi B$$

For the purposes of this course, we will *define* optical depth, transmittance etc. assuming the diffuse approximation when dealing with infrared radiation, and assuming the direct-beam case when dealing with incoming solar radiation. So you don't need to worry about the distinction.

As you work through the problem set, you will show that upward irradiance at the top of the atmosphere is (dropping the ν suffixes for clarity, but remember this is all specific to a particular frequency)

$$I_0 = I_s e^{-\chi_s} + \pi \int_0^{\chi_s} B(\chi) e^{-\chi} d\chi$$

or

$$I_0 = I_s \mathcal{T}(\chi_s) + \pi \int_{\chi=0}^{\chi_s} B(\chi) \mathcal{T}(\chi) d\chi$$

where I_s is the upward irradiance at the surface and $B(\chi)$ is the Planck function at optical depth χ. The first term on the RHS corresponds to irradiance from the surface, reduced by the total transmittance of the air column, while the second term corresponds to contributions to irradiance from intervening "slabs" of atmosphere of thickness $d\chi$.

The crucial piece of physics is that doubling the concentration of CO_2 approximately doubles $\chi_\nu(z)$ at any given altitude for any frequency ν in which atmospheric absorption is dominated by CO_2, independent of what $\chi_\nu(z)$ was to begin with.

Since the scale height of $\chi(z)$ is approximately independent of the concentration of CO_2 (it's a trace gas, so injecting more doesn't change $p(z)$ or $T(z)$), if you understand how the function $I_0(\chi_s)$ depends on χ_s, the optical depth at the surface at frequency ν, then you've understood the CO_2 greenhouse effect.

We can solve Schwarzschild's equation numerically to obtain $I_0(\chi_s)$ with the above idealised temperature profile, to give results that are remarkably close to a full radiative transfer calculation (confirming the assumption of linear pressure broadening is reasonably good across a broad range of wavelengths). Note the broad region between $\chi_s = 2$ and $\chi_s = 20$ over which $I_0 \propto \ln(\chi_s)$: these are the optical depths at which I_0

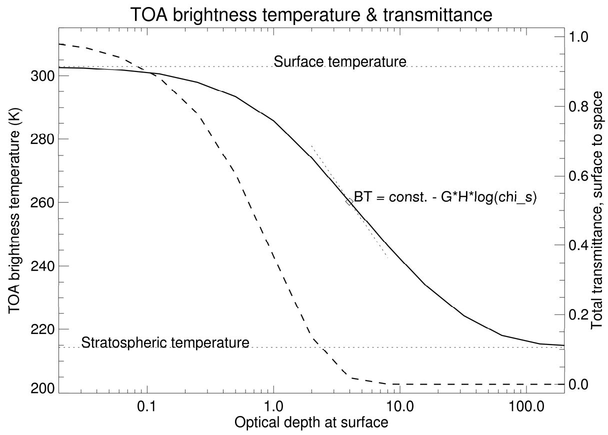

Figure 4: Calculated TOA irradiance (expressed as a brightness temperature) as a function of optical depth to the surface for a constant tropospheric lapse rate, isothermal stratosphere and linear pressure broadening. Dashed line and right hand axis shows total transmittance. Symbol and dotted line show predicted linear relationship $T_{\mathrm{BB}} = \mathrm{const} - \Gamma H_\chi \ln(\chi_{\mathrm{s}})$ in the "high tropopause limit."

shows most sensitivity to doubling CO_2.

You might well argue we are now done, but the Oxford tradition is that you haven't understood something until you can solve it analytically, so we consider some limits...

Strong and weak absorption:

If χ_1 is the optical depth at the tropopause, we consider three cases:

1. Optically thin stratosphere $\chi_1 \ll 1$ and optically thin troposphere $\chi_{\mathrm{s}} - \chi_1 \ll 1$.
2. Optically thick stratosphere $\chi_1 \gg 1$ and optically thick troposphere $\chi_{\mathrm{s}} - \chi_1 \gg 1$.
3. Optically thin stratosphere $\chi_1 \ll 1$ and optically thick troposphere $\chi_{\mathrm{s}} - \chi_1 \gg 1$.

The third case is only approximate because in the Earth's atmosphere the optical depth of the tropopause is around 5-10% of the optical depth at the surface, but it gives the right functional form of $I_0(\chi_{\mathrm{s},\nu})$.

Case 2: Strong absorption limit.

If the surface radiates as a black body, you can re-express I_0 as follows

$$I_0 = \pi B(0) + \pi \int_0^{\chi_{\mathrm{s}}} \frac{dB(\chi)}{d\chi} \mathcal{T}(\chi)\, d\chi$$

This helps understand the very strong absorption limit (case 2). In this case all transmittances are small and upward irradiance is dominated by the Planck function in the stratosphere (assumed isothermal), which doesn't change when we increase CO_2 in this simple model.

Case 1: Weak absorption limit.

Equivalently,

$$I_0 = \pi B(\chi_{\mathrm{s}}) - \pi \int_0^{\chi_{\mathrm{s}}} \frac{dB(\chi)}{d\chi} \mathcal{A}(\chi) d\chi \ ,$$

where $\mathcal{A}$ is the absorptance. This is relevant to the weak absorption limit (case 1). As $\mathcal{A}(\chi\mathrm{s}) \to 0$, the integral is dominated by $dB(\chi)/d\chi$ near the surface, so $I_0(\chi_{\mathrm{s}})$ is linear in total atmospheric absorptance $\mathcal{A}(\chi\mathrm{s})$ and hence linear in χ_{s}. This limit applies to CFCs, which is why their radiative impact is linear in CFC concentration.

Note that, in this limit, $dI_0/d(\ln(\chi_{\mathrm{s}})) \propto \chi_{\mathrm{s}}$ and is hence also small: doubling CO_2 has little impact at wavelengths far from the main CO_2 absorption features.

Case 3: strongly absorbing troposphere, weakly absorbing stratosphere

Between these two limits, peak sensitivity to doubling CO_2 must occur in a region were $I_0 \propto \ln(\chi \mathrm{s})$ by simple geometry. If you neglect the stratosphere and assume $\chi_\mathrm{s} \gg 1$ (perhaps best thought of as the "high tropopause" limit), you can even get more-or-less the right constant of proportionality: CO_2 doubling reduces the top-of-atmosphere "brightness temperature" (the temperature a black body would need to have to generate the same irradiance at a given wavelength) by $\Gamma H_\chi \ln(2)$, in reasonable agreement with MODTRAN. So you now understand the enhanced greenhouse effect due to increasing CO_2.

Note: some (even quite sophisticated) textbooks will tell you the logarithmic dependence of outgoing long-wave radiation on CO_2 concentration arises from the shape of the CO_2 absorption feature, which makes it sound rather accidental. I hope I have convinced you that it can be understood at an individual-wavenumber level. This doesn't mean I disagree with these textbooks, because the shape of the CO_2 absorption feature also arises from the basic physics of Schwarzschild's equation, the lapse rate and the exponential decay of optical depth with height.